APPENDIX E

HURRICANE GENERATED SURFACE WAVES

E-1. General. In the prediction of short period surface waves generated by hurricane winds, the determination of the wave fetch and duration from a wind field is more difficult than for more normal weather conditions. Difficulties arise because hurricane winds blow spirally inward toward the center as the storm system moves over the ocean. Fetch areas in which the wind speed and direction remain approximately constant are always small and a fully arisen sea would seldom be realized. A mathematical model developed for estimating wave characteristics in a hurricane is described in the Shore Protection Manual (1977). This model provides a reasonable approach for predicting hurricane waves provided that the model is modified to account for the more recent SPH wind field criteria that were presented in Appendix C. The original model together with the necessary modifications are described herein and an example problem is given to illustrate the computational procedures for predicting hurricane waves.

E-2. Prediction Method. For a moving hurricane, the following expressions can be used to obtain an estimate of the deep water significant wave height and period at the point of maximum wind:

$$H_o = 16.5 \exp\left(\frac{R\ \Delta p}{100}\right)\left(1 + \frac{0.208\ \kappa\ V_f}{V_{xm}^{1/2}}\right) \qquad [E-1]$$

$$T_o = 8.6 \exp\left(\frac{T\ \Delta p}{200}\right)\left(1 + \frac{0.104\ \kappa\ V_f}{V_{xm}^{1/2}}\right) \qquad [E-2]$$

where

$\exp(x)$ = e^x, in which e = 2.71828. . . . . .

H_o = deep water significant wave height in feet

T_o = the corresponding significant wave period in seconds

R = radius of maximum winds in nautical miles

Δp = $p_n - p_o$ in inches of mercury in which p_n is the peripheral pressure and p_o is the central pressure

V_f = forward speed of hurricane in knots

V_{xm} = maximum sustained wind speed in knots at radius R.

$$V_{xm} = 0.9 \ V_{gx} + 1.5 \ V_f^{0.63} \cos \beta \qquad [E-3]$$

and for the PMH

$$V_{xm} = 0.95 \ V_{gx} + 1.5 \ V_f^{0.63} \cos \beta \qquad [E-4]$$

β = angle between the hurricane track direction and the maximum surface wind direction

V_{gx} = maximum gradient wind speed, see Equation [C-2]

$$V_{gx} = K \ (\Delta p)^{1/2} - \frac{Rf}{2} \qquad [E-5]$$

K = a coefficient that depends on the air density just above the sea surface and can be obtained from either Figure C-9 or C-10 for the SPH and PMH, respectively

f = Coriolis parameter = $2 \ \omega \sin \phi$ in which ϕ is the geographical latitude of the hurricane eye

ω = $2\pi/24$ radians per hour

κ = a coefficient depending on the forward speed of the hurricane and the increase in effective fetch due to the hurricane translation. For a slow moving hurricane it is suggested that $\kappa = 1.0$.

a. After determination of H_o for the point of maximum wind it is possible to obtain the approximate deepwater wave height H_o for other points within the hurricane by use of Figure E-1. A sufficient approximation of the deepwater significant wave period can be obtained from

$$T_o = 2.13\ H_o^{1/2} \qquad [E-6]$$

in which H_o is in feet. The latter expression is derived from empirical data which show that the wave steepness H/T^2 will be about 0.22.

*********************** EXAMPLE PROBLEM ***********************

Given: Consider a SPH at latitude 37 degrees N situated over the ocean (mile post about 2300 n mi.) with R = 34 nautical miles and V_f = 26 knots.

Find: The deepwater significant wave height and period.

Solution: The Coriolis parameter is given by

$$f = 2\ \omega \sin \phi = 2\left(\frac{2\pi}{24}\right) \sin 37 = 0.315\ .$$

For a SPH, P_n = 29.77 inches.

At the mile post given the central pressure p_o, according to Figure C-1, is approximately 27.65 inches. Therefore,

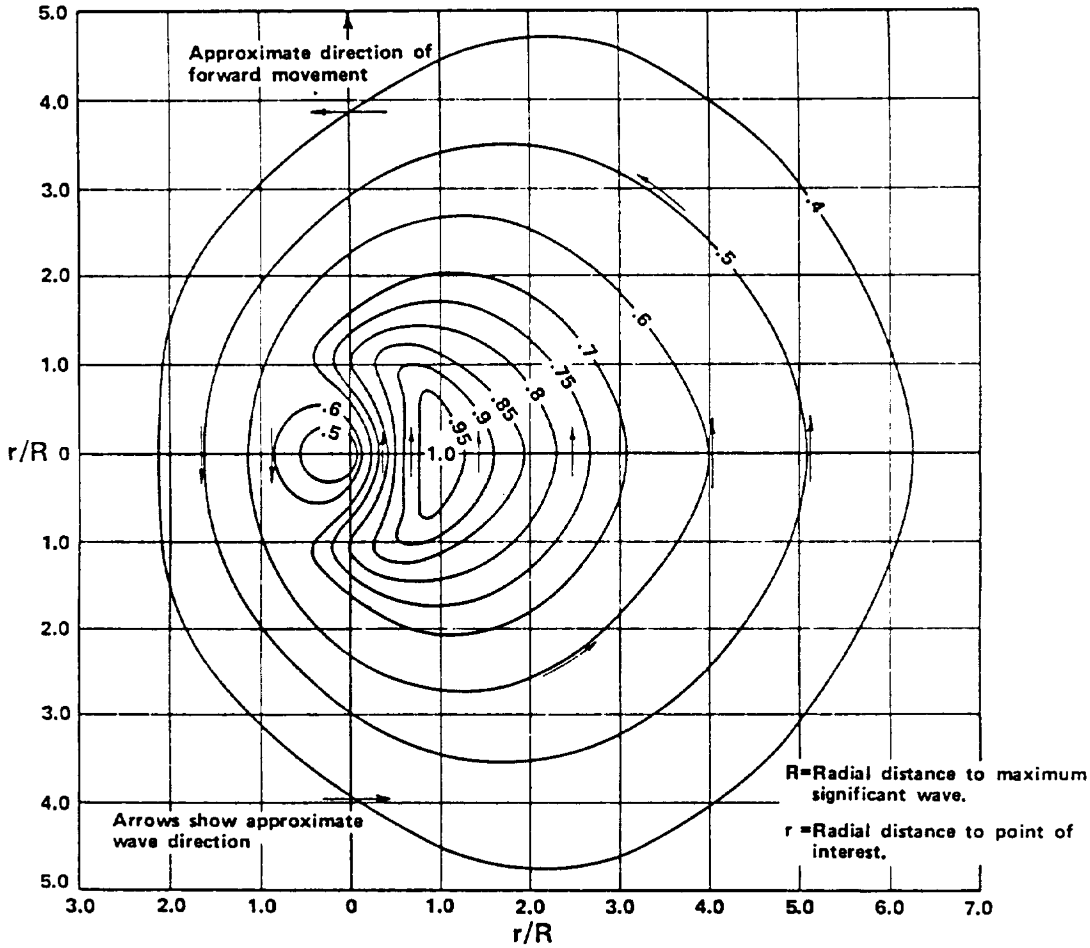


Figure E-1. Isolines of relative significant wave height for slow-moving hurricane.

$$\Delta p = 29.77 - 27.65 = 2.12 \text{ inches.}$$

The coefficient K found from Figure C-9 is about 66.6 knots-inches. Using Equation E-5

$$V_{gx} = K \, \Delta p^{1/2} - \frac{rf}{2} = 66.5 \, (2.12)^{1/2} - \frac{35(0.315)}{2} = 91.3 \text{ knots.}$$

Using Equation [E-3] and assuming that the maximum wind is blowing in a direction parallel to the hurricane path (β = 0), then

$$V_{xm} = 0.9 \, (91.3) + 1.5 \, (26)^{0.63} \, (1) = 93.9 \text{ knots.}$$

Assume for simplicity that κ = 1. Using Equation [E-1]

$$H_o = 16.5 \exp\left(\frac{R \, \Delta p}{100}\right) \left(1 + \frac{0.208 \, \kappa \, V_f}{V_{xm}^{1/2}}\right)$$

$$H_o = 16.5 \exp\left[\frac{35(2.12)}{100}\right] \left[1 + \frac{0.208 \, (1) \, (26)}{(93.9)^{1/2}}\right] \simeq 54.0 \text{ feet.}$$

Using Equation [E-2]

$$T_o = 8.6 \exp\left(\frac{R \, \Delta p}{200}\right) \left(1 + \frac{0.104 \, \kappa \, V_f}{V_{xm}^{1/2}}\right)$$

$$T_o = 8.6 \exp\left[\frac{(35) \, (2.12)}{200}\right] \left[1 + \frac{0.104 \, (1) \, (26)}{(93.9)^{1/2}}\right] = 15.9 \text{ sec .}$$

Using Equation [E-6]

$$T_o = 2.13 \, (H_o)^{1/2} = 2.13 \, (54)^{1/2} = 15.7 \text{ sec}$$

which shows that the latter simple relation is of sufficient accuracy and will be used in subsequent calculations. With a knowledge of the deepwater significant wave height and period it is possible to determine the changes in these wave characteristics as the hurricane moves over the continental shelf. In order to make this determination it is necessary to account for the combined effects of bottom friction, refraction, the continued action of the wind and the forward speed of the hurricane. It is necessary to use relatively short wind fetch length due to the revolving winds and thus a numerical integration procedure must be utilized. When waves refract over the bottom contours, it will be also necessary to use appropriate refraction diagrams as presented in the Shore Protection Manual (1984). In the use of the numerical procedure, an effective fetch length F_e is required which is given by

$$F_e = \left(\frac{H_o}{0.0555\ V_{xm}} \right)^2 . \qquad [E-7]$$

For the preceding example problem

$$F_e = \left[\frac{54.0}{(0.555)(93.9)} \right]^2 = 107.4 \text{ n mi.}$$

The deepwater significant wave height H_o can be evaluated by using a modified version of Equation [E-7] as given by

$$H_o = 0.0555\ V_{xm}\ (F'_e + \Delta F)^{1/2} \qquad [E-8]$$

in which F'_e is defined in the procedures outlined below and ΔF is a specified fetch length interval used in the numerical integration technique. The procedure for calculating the surface waves over the continental shelf is illustrated by using a bottom profile seaward of the Chesapeake Bay entrance (profile taken from the 1977 Shore Protection Manual) and the hurricane taken from the previous example. The water depths used are assumed to include the storm surge and astronomical tide over the shelf area. Refraction is neglected in this example. The result of the computations are shown in Table E-1. Explanation of the computations are as follows:

Column 1 of Table E-1 is the distance in nautical miles measured seaward from the entrance to Chesapeake Bay, using increments of 5 n mi. for each section.

TABLE E-1. Computation Of Wind Waves Over Continental Shelf

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
X	d_1	d_2	$\bar{d}_t$	F_e	H_o	T_o	$\frac{\bar{d}_t}{L_o}$	$\frac{f_f H_o \Delta X}{(d_t)^2}$	K_f	H_o'	F_e'	T_o'	$\frac{d_2}{L_o'}$	K_s	H
65	1004	504	754	107.4	54.0	15.7	0.597	0.029	1.000	54.0	107.4	15.7	0.399	0.9758	52.7
60	504	194	349	107.4	54.0	15.7	0.277	0.135	0.985	53.2	104.2	15.5	0.158	0.9130	48.6
55	194	158	176	107.4	54.0	15.7	0.139	0.530	0.930	50.2	92.9	15.1	0.135	0.9156	46.0
50	158	122	140	97.9	51.6	15.3	0.117	0.800	0.900	46.4	79.2	14.5	0.113	0.9239	42.9
45	122	116	119	84.2	47.8	14.7	0.108	1.026	0.850	40.6	60.8	13.6	0.123	0.9192	37.3
40	116	116	116	65.8	42.3	13.9	0.118	0.956	0.890	37.6	52.1	13.1	0.133	0.9161	34.4
35	116	110	113	57.1	39.4	13.4	0.123	0.938	0.880	34.7	44.3	12.5	0.136	0.9154	31.8
30	110	88	99	49.3	36.6	12.9	0.116	1.135	0.870	31.8	37.3	12.0	0.119	0.9209	29.3
25	88	78	83	42.3	33.9	12.4	0.105	1.496	0.830	28.1	29.1	11.3	0.119	0.9209	25.9
20	78	68	73	34.0	30.4	11.7	0.103	1.734	0.810	24.6	22.3	10.6	0.120	0.9204	22.6
15	68	62	65	27.3	27.2	11.0	0.106	1.957	0.790	21.5	17.0	9.9	0.124	0.9189	19.8
10	62	52	57	22.0	24.4	10.5	0.101	2.283	0.750	18.3	12.3	9.1	0.123	0.9192	16.8
5	52	44	48	17.3	21.7	9.9	0.095	2.863	0.720	15.6	9.0	8.4	0.121	0.9200	14.4
0	44	36	40	14.0	19.4	9.4	0.088	3.686	0.650	12.6	5.9	7.6	0.122	0.9196	11.6

Column 2 is the depth d_1 at the beginning of each section.

Column 3 is the depth d_2 at the shoreward end of each section.

Column 4 is $\bar{d}_t$ the average of Columns 2 and 3 to the nearest foot.

Column 5 is the effective fetch F_e in nautical miles, and is determined for the first step directly from Equation [E-7]. For successive steps, $F_e = F'_e + \Delta F \leq 54.9$ n mi. where F'_e is given in Column 12 one line above in each case, and $\Delta F = 5$ n mi.

Column 6 is the deepwater significant wave height H_o and is obtained from Equation [E-1] on the first step and Equation [E-8] for the succeeding steps.

Column 7 is the deepwater significant wave period and is obtained by Equation [E-6].

Column 8 is the average water depth $\bar{d}_t$ divided by the deepwater wave length L_o where

$$\frac{\bar{d}_t}{L_o} = \frac{2\pi \, \bar{d}_t}{gT_o^2} \quad .$$

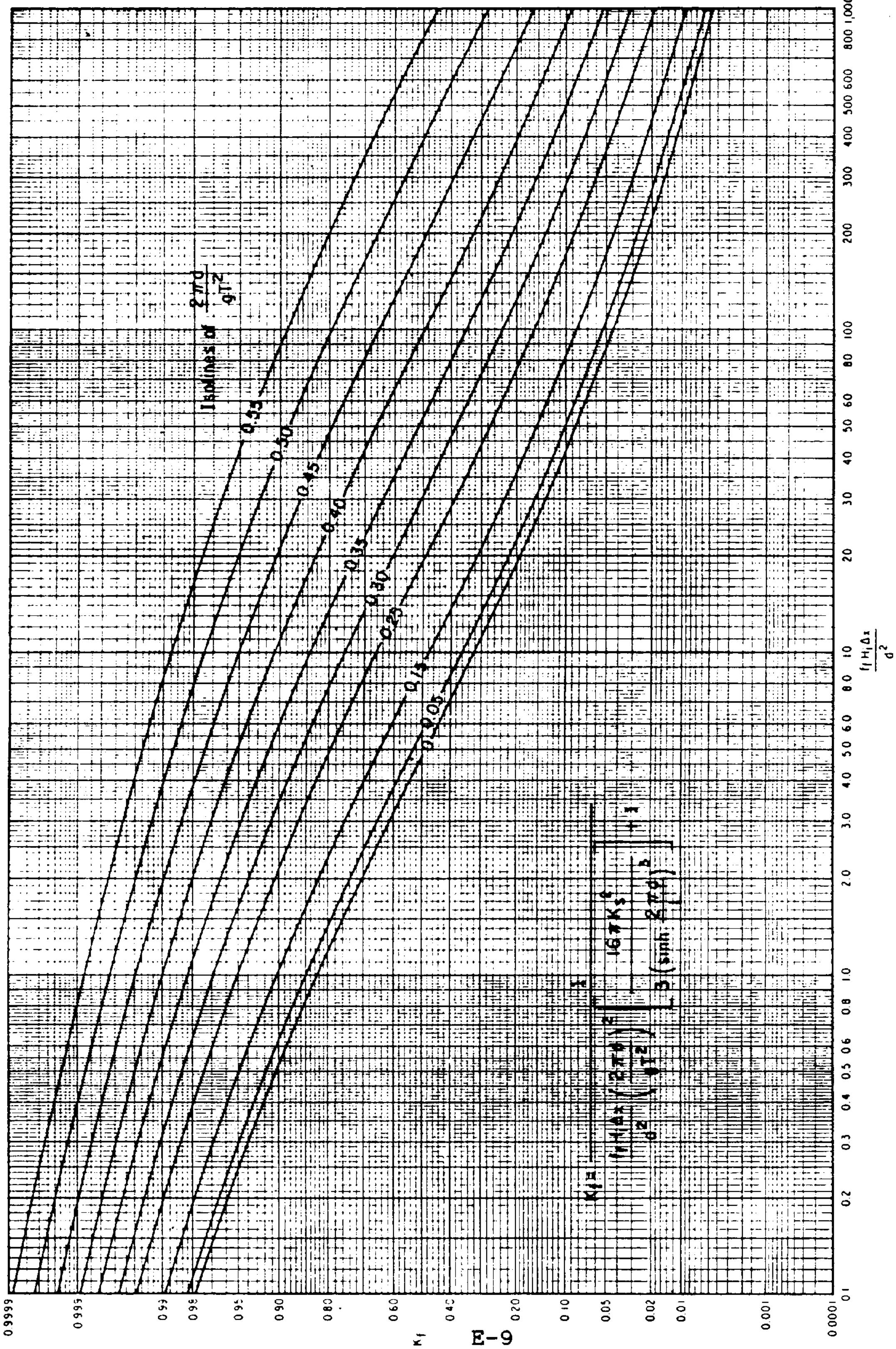
Column 9 is the parameter in Figure E-2

$$\frac{f_f \, H_i \, \Delta X}{d^2} = \frac{f_f \, H_o \, \Delta X}{(\bar{d}_t)^2}$$

in which friction factor f_f is assumed to be 0.01 ΔX = 5 (6080) = 30,400 feet.

Column 10 is the friction factor K_f which is obtained from Figure E-2 using column 8 and column 9.

Column 11 is the equivalent deepwater wave height H'_o and is obtained from $H'_o = K_f \, H_o$ (the products of column 6 and 10).


Isolines of $\frac{2\pi d}{gT^2}$
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.15
0.05
K_t
$\frac{f_t H_i \Delta x}{d^2}$

Column 12 is the equivalent effective lenght F_e and is obtained from Equation [E-7] by replacing H_o by H'_o or

$$F'_e = \left(\frac{H'_o}{0.0555\ V_{xm}}\right)^2 \quad .$$

Column 13 is the equivalent deepwater wave period T_o in seconds and computed by

$$T'_o = 2.13\ (H'_o)^{1/2} \quad .$$

Column 14 is the depth d_2 divided by the equivalent deep water wave length

$$L_o = 5.12\ (T'_o)^2 \quad .$$

Column 15 is the shoaling coefficient K_s which can be obtained from Table C-1, Appendix C of the Shore Protection Manual in which $H/H'_o = K_s$ as a function of d_2/L_o (column 14).

Column 16 is the significant wave height H in which

$$H = K_s\ H'_o$$

or the products of Columns 11 and 15.

After completing the computations for the first row, the computations are commenced on the second row etc., until the table is completed.